## 3.3 Computational Rules of Propositional Logic

Let Theorem: Rules of Calculation of Propositional Logic1. PAssociative laws:, Q, and R be logical expressions. Then the following calculation rules apply:

Commutative laws: PP QQ RR PP QQ RR

Distribution laws: PP QQ QQ PP

Proof: PP QQ RR PP QQ PP RR

The proof of the associative laws and the commutative laws is left to you for practice. By using truth tables, we show that the distribution laws apply:

This shows that P ∧ (Q ∨ R) ≡ (P ∧ Q) ∨ (P ∧ R) applies. □

The following laws of the British mathematician Augustus de Morgan formulate a connec-This shows that P ∨ (Q ∧ R) ≡ (P ∨ Q) ∧ (P ∨ R) applies. □

tion between the negation and the conjunction or disjunction.

Let andTheorem: Laws of de MorganP and Q be propositional formulas. Then it applies that:¬ P∧Q ≡ ¬P∨ ¬Q

Proof: ¬ P∨Q ≡ ¬P∧ ¬Q

We prove the statements by applying truth tables. It is:

The proof of the second law is Please note that the laws of de Morgan can be generalized.left to you for practice. □

Theorem: Generalization of the Laws of de MorganLet P1, ..., Pn be propositional expressions. It applies that:¬P1 ∧P2 ∧…∧Pn≡ ¬P1 ∨ ¬P2 ∨…∨ ¬Pn

Proof:and ¬P1 ∨P2 ∨…∨Pn ≡ ¬P1 ∧ ¬P2 ∧…∧ ¬Pn

Because conjunction and disjunction are associative and because the laws of de Morgan

apply, it follows that:and ¬P ≡≡¬P1 ¬ P2 … P − ∧∨PPn¬nPn

¬P ∨ ¬P ∨…∨ ¬Pn 1

In addition to the laws of de Morgan and the associative, commutative, and distributive¬P ≡≡¬P11 ∧ ¬P22 ∧…∧ ¬Pnn−−11 ∧Pn¬nPn

laws, there are a number of other important calculation rules.

Let P and Q be logical expressions. Then the following rules apply: P ∧ ¬P ≡ 0¬¬P ≡ PP ∧ 1 ≡ PP ∨ P ≡ PP ∨ 1 ≡ 1 Theorem: Further Rules of Propositional Logic 2. Neutrality of conjunction: P ∧ 0 ≡ 0P ∨ 0 ≡ PP ∧ P ≡ P

1. Identity of the conjunction:

Neutrality of the disjunction:

Identity of the disjunction:

Double negation:

Idempotence of conjunction: 9. Tautology: P ∨ ¬P ≡ 1 P ∨ (P ∧ Q) ≡ PP ∧ (P ∨ Q) ≡ PP ⇒ Q ≡ ¬P ∨ QP ⇔ Q ≡ (P ∧ Q) ∨ (¬P ∧ ¬Q)P ⇔ Q ≡ (¬P ∨ Q) ∧ (P ∨ ¬Q)P ∨ (¬P ∧ Q) ≡ P ∨ QP ∧ (¬P ∨ Q) ≡ P ∧ Q

Idempotence of disjunction:

Contradiction:

Absorption with respect to conjunction:

Absorption with respect to disjunction:

Absorption with respect to conjunction with negation:

Absorption with respect to disjunction with negation:

Conversion of the implication into a disjunction:

Transformation of equivalence into a conjunction:

Transformation of equivalence into a disjunction:

Proof:

The proofs for rules 1—4 are left to you as an exercise.

The proofs for rules 8 and 9 (contradiction and tautology) we have already given above. We’ll show the remaining statements with the help of truth tables:

Rule 5:

It follows that ¬¬P ≡ P. □ Rule 6:

It follows that P ∧ P ≡ P. □

Rule 7:

It follows that P ∨ P ≡ P. □ Rule 10:

It follows that P ∧ (P ∨ Q) ≡ P. □ Rule 11:

It follows that P ∨ (P ∧ Q) ≡ P. □

Rule 12:

It follows that P ∧ (¬P ∨ Q) ≡ P ∧ Q. □

Rule 13:

∨ It follows that P ∨ (¬P ∧ Q) ≡ P ∨ Q. □ Rule 14:

It follows that P ⇒ Q ≡ ¬P ∨ Q. □ Rule 15:

It follows that P ⇔ Q ≡ (¬P ∨ Q) ∧ (P ∨ ¬Q). □

Rule 16:

)

It follows that P ⇔ Q ≡ (P ∧ Q) ∨ (¬P ∧ ¬Q). □